Equations

Astronomy 160: Stellar Astrophysics

EQUATIONS

1 Basic Gravity

Gravitational Force: $F = GM_1M_2/r^2$
Grav. Potential Energy between point particles: $\Omega = -GM_1M_2/r$
Kepler's Third Law:$P^2 = \frac{4\pi^2}{G(M_1+M_2)}a^3$
Virial Theorem:$< U_{th} >= -1/2\Omega_{grav}$
Free-fall time:$t_{ff} \approx \sqrt{3\pi/(32G\rho)}$

2 Magnitudes and Light

$$m_1 - m_2 = -2.5\log_{10}(\frac{F_1}{F_2})$$

Distance modulus: $m - M = 5\log_{10} d - 5$
distance in pc:$d = 1/\pi''$
Wien's law:$\lambda_{max}T = 2.898 \times 10^{-3}$mK
Blackbody Radiation Flux: $F_{surf} = \sigma T^4$
Blackbody Energy Density:$U = aT^4$
Planck Fn.: $B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT}-1}$
$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$
Photons: $E = h\nu$ and $p = E/c$
Doppler Effect (v≪c): $\frac{\lambda_{\rm obs}-\lambda_{\rm rest}}{\lambda_{\rm rest}} = v_r/c$
Doppler Effect (v∼c):$\nu_{\rm obs} = \frac{\nu_{\rm rest}\sqrt{1-v^2/c^2}}{1+v_r/c}$
Hydrogen energy levels: $E_n = -\frac{\mu e^4}{2\hbar^2 n^2}$ (cgs) $= -13.6$ eV$/n^2$

$$g_n = 2n^2$$

(degeneracy of levels in H)

3 Hot Gas

Ideal gas law: $P_{\text{gas}} = nkT = \rho kT/(\mu m_H)$
Maxwell-Boltzmann Eqn: $n_v \mathrm{d}v = n(m/2\pi kT)^{3/2} e^{-1/2mv^2/kT} 4\pi v^2 \mathrm{d}v$
$3/2kT = 1/2mv_{\text{rms}}^2$
1-D Maxwell-Boltzmann: $n_{v_x} \mathrm{d}v_x = n(m/2\pi kT)^{1/2} e^{-mv_x^2/2kT} dv_{\text{x}}$
Boltzmann Eqn: $N_j/N_i = \frac{g_j}{g_i} e^{-(E_j - E_i)/kT}$
Saha Equation:$\frac{n_{i+1}}{n_i} = \frac{2U_{i+1}}{n_e U_i} (\frac{(2\pi m_e kT)^{3/2}}{h^3}) e^{-E_i/kT}$
Partition Function: $U = \sum g_i e^{-E_i/kT}$

4 Radiation Transfer

$I = I_o e^{-\tau}$
$\tau = \int \kappa\rho dx = \int n\sigma dx$
Equation of Transfer: $\mathrm{d}I = j_\lambda \rho \mathrm{d}s - \kappa_\lambda \rho I \mathrm{d}s$
Solution to Equation of Transfer:

$$I_\nu(\theta) = I_{0,\nu}(\theta) e^{-\tau/cos\theta} + \int S_\nu(t) e^{-t/cos\theta} dt/cos\theta$$

Hydrostatic Equilibrium: $dP/dr = -GM_r \rho/r^2$

Radiation Transport of energy: $dT/dr = -\frac{3}{16\sigma} \frac{\kappa\rho}{T^3} \frac{L_r}{4\pi r^2}$

5 Stellar Atmospheres and Spectra

Opacity of an atomic line: $k_\nu \rho = \frac{\pi e^2}{m_e c} f_{ij} \phi_\nu n_i (1 - e^{\frac{-h\nu}{kT}})$
Lorentz Profile: $\phi_\nu = \frac{\gamma/4\pi^2}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2}$
Line Depth: $A_\lambda = 1 - \frac{F_\lambda}{F_c}$

6 Stellar Structure

Hydrostatic Equilibrium: $dP/dr = -GM_r\rho/r^2$
Mass Conservation: $dM_r/dr = 4\pi r^2\rho$
Energy Generation: $dL_r/dr = 4\pi r^2\rho\epsilon$
Radiation Transport of energy:$dT/dr = -\frac{3}{16\sigma}\frac{\kappa_R\rho}{T^3}\frac{L_r}{4\pi r^2}$
Adiabatic thermodynamics: $PV^\gamma = \text{const}$
Polytropes and Lane-Emden Eqn: Eqn. of State: $P = K\rho^{\frac{1+n}{n}}$
Lane-Emden Eqn:

$$\frac{1}{\xi^2}d/d\xi(\xi^2 d\theta/d\xi) = -\theta^n$$

where $\rho(r) = \rho_c\theta^n(r)$ and $\xi = r/a$

7 Nuclear Reactions

Collision (or reaction) rate between particles x and i: $r = n_x n_i \sigma v$
pp-chain first step: $H + H - - >^2 H + e^+ + \nu_e$

8 Constants

c = 2.99792 × 10^10 m/s
h = 6.623 × 10^{-27} erg s
$\sigma = 5.67 \times 10^{-5}\ erg/cm^2K^{-4}$
$a = 7.56 \times 10^{-16}Jm^{-3}K^{-4}$
$L_\odot = 3.90 \times 10^{33}$ W
$M_\odot = 1.99 \times 10^{33}$ g
$G = 6.67 \times 10^{-11}Nm^2/kg^2$
$k = 1.38 \times 10^{-23}J/K$
$1eV = 1.6 \times 10^{-19}J$
Units: 206265 AU = 1 parsec (pc)